Plane Trigonometry. By S. L. LONEY, M. A., Late Fellow of Sidney Sussex College, Cambridge, Professor at the Royal Holloway College. Macmillan & Co.

This work is divided into two parts; the subject of Part I is Geometrical Trigonometry, and that of Part II Analytic Trigonometry. Part I includes the subjects that are usually presented in elementary trigonometry. The trigonometrical functions of angles are defined as ratios. Before introducing, however, the idea of positive and negative angles and directed lines, the author establishes many of the relations between the functions of acute angles, and employs the natural functions of special angles in the computation of heights and distances, thus introducing the reader to one of the most important and interesting applications of the science. The geometrical proofs of the more common formulas are full, clear, and rigorous. The proving of trigonometrical identities, and the solution of trigonometrical equations receive the attention which their importance warrants. The examples in these subjects as in all others are numerous and well chosen. The properties of triangles and their connected circles are fully discussed.

In Part II the author first deduces the common exponential and logarithmic series, and then treats of complex quantities under their trigonometric form. He establishes De Moivre's Theorem and applies it to finding any sort of a complex quantity and to the expansion of cos $n\theta$ and sin $n\theta$ in terms of the functions of θ. From these general formulas are obtained the common expansions of sin θ and cos θ. Next follows the expansions of the cosine and sine of an angle in terms of the cosines and sines of multiples of that angle, also the expansions of the cosine and sine of a multiple angle in terms of the powers of the cosines and sines of the angle. The author treats with marked clearness the exponential series for complex quantities, the trigonometric functions of complex angles, and the hyperbolic functions defined analytically. It would have added to the interest and clearness of view of the reader to have here compared the geometrical representations of the trigonometric and hyperbolic functions. The discussion of the many-valued logarithms of negative and complex numbers is very satisfactory. Among the other subjects treated the most important are the value of π, summation of series, expansion in series, factoring of mathematical expressions, proportional parts, errors of observation, solution of cubic equations, and geometric representation of complex quantities. The author has succeeded in his purpose to produce "a fairly complete elementary text-book on Plane Trigonometry." The faithful student of this treatise "will have little to unlearn when he commences to read treatises of a more difficult character." The style is clear

and simple; even when it is diffuse, the author never hides his thoughts with words either large or small. It is a work that will well repay the reading. The typography of the book is excellent.

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Uniplanar Algebra. Being Part I of a Propædeutic to the Higher Mathematical Analysis. By IRVING STRINGHAM, Ph. D. San Francisco: The Berkeley Press, 1893. pp. xii+141.

If any one expects to find in this little book a text-book on algebra like, except in name, to most text-books on that subject he will be disappointed. The book is not a beginner's book; it is elementary only in so far as it begins at the beginning.

Starting with the theory of proportion as stated by Euclid, the author builds upon this the algebra of real quantities and establishes the laws of combination of such quantities by simple geometrical constructions. After devoting a chapter to the definition and discussion of the circular and hyperbolic functions he takes up the algebra of complex quantities. By means of Argand's mode of representation he shows that the laws which were established for real apply as well to complex quantities. At the end of this chapter he states briefly and clearly the characteristics of a logically complete algebra, and incidentally points out that an algebra which "admits evolution and the logarithmic process, but precludes the imaginary and the complex quantity is logically only the fraction of an algebra."

Then follow three chapters devoted respectively to generalized circular and hyperbolic functions, to graphical transformations and to the properties of polynomials. The first two of these, though interesting in themselves, are digressions from the main argument and might perhaps be omitted without serious injury to the book. The third, however, is more important, for it contains a proof of the so-called fundamental theorem of algebra, viz: that every algebraic equation has a root, a theorem which in most text-books is not proved and in many is totally ignored.

In his preface Professor Stringham calls attention to the fact that algebra, unlike geometry, which is a model of exact reasoning, has become "a collection of processes practically exemplified and of principles inadequately explained." He has endeavored in his book to do just the reverse and to give to his readers the theory and not the practice of algebra. In our opinion he has succeeded exceedingly well. The first four chapters give a complete and well-reasoned account of the fundamental principles of algebra. Moreover, the book is interesting from the fact that it contains things not found in the ordinary text-books. Such for example are: Euclid's theory of proportion, Napier's definition of